Visualization, Identification, and Estimation in the Linear Panel Event-Study Design

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Motivation

- Event studies and related methods increasingly popular in applied micro

Source: Currie et al. (2020) Figure 4

Today

1. Provide suggestions on the construction of event-study plots
   - Packages `xtevent` in Stata and `eventstudyr` in R facilitate adoption
2. Review approaches to identification and their economic content
3. Illustrate the performance of different estimators under some economically reasonable data-generating processes

Setup

Data

- Units $i \in \{1, ..., N\}$, e.g., states
- Periods $t \in \{1, ..., T\}$, e.g., years
- Scalar outcome y_{it}, e.g., employment
- Scalar policy z_{it}, e.g., minimum wage

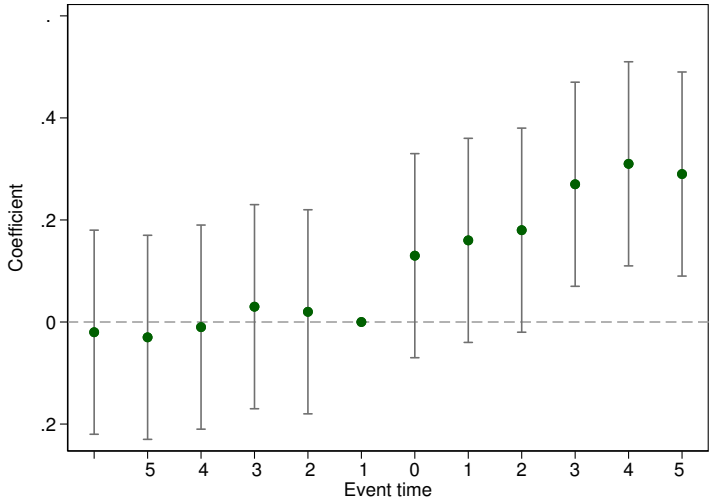
Linear Panel Model

$$y_{it} = \alpha_i + \gamma_t + q_{it}'\psi + \sum_{m=-G}^{M} \beta_m z_{i,t-m} + C_{it} + \varepsilon_{it} \qquad \text{(linear panel model)}$$

- Unit fixed effects α_i and time fixed effects γ_t
- Observed controls q_{it}
- Unobserved confound C_{it} potentially related to policy z_{it}
- Unobserved error ε_{it} unrelated to policy z_{it}
- Parameters of interest $\{\beta_m\}_{m=-G}^{M}$
  - No *ceteris paribus* effect of policy more than G periods in the past or M periods in the future

Event-study Plots

Typical Event-study Plot



Building the plot

$$y_{it} = \alpha_i + \gamma_t + q'_{it}\psi + \sum_{m=-G}^{M} \beta_m z_{i,t-m} + C_{it} + \varepsilon_{it} \qquad \text{(linear panel model)}$$

For the event-study plot we want to:

- Show cumulative effects of the policy $\rightarrow$ replace z_{it} with Δz_{it}
- Show pre-G and post-M dynamics $\rightarrow$ add L_G extra leads and L_M extra lags

Estimating Equation

$$y_{it} = \sum_{k=-G-L_G}^{M+L_M-1} \delta_k \Delta z_{i,t-k} + \delta_{M+L_M} z_{i,t-M-L_M} + \delta_{-G-L_G-1}(-z_{i,t+G+L_G}) + \alpha_i + \gamma_t + q_{it}'\psi + C_{it} + \varepsilon_{it}$$

(estimating equation)

- Will refer to index k as *event time*
- Will refer to vector δ as *event time path* of outcome

Interpretation under staggered adoption

$$\ldots \sum_{k=-G-L_G}^{M+L_M-1} \delta_k \Delta z_{i,t-k} + \delta_{M+L_M} z_{i,t-M-L_M} + \delta_{-G-L_G-1}(-z_{i,t+G+L_G}) \ldots$$

(key part of estimating equation)

Say that for each unit i, z_{it} starts at 0 and switches to 1 at time $t^*(i)$. Then:

$$\begin{aligned} \Delta z_{i,t-k} &= \mathbf{1}\{t^*(i) = t-k\} \\ z_{i,t-M-L_M} &= \mathbf{1}\{t^*(i) \leq t-M-L_M\} \\ 1 - z_{i,t+G+L_G} &= \mathbf{1}\{t^*(i) > t+G+L_G\} \end{aligned}$$

Interpetation as cumulative effects of policy

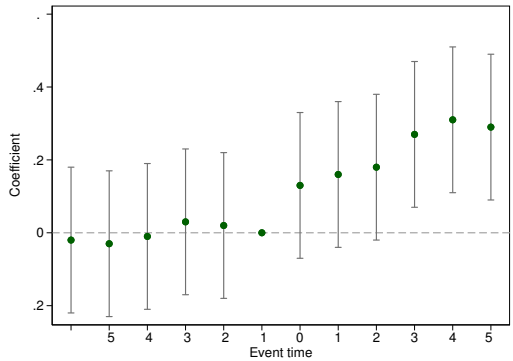
$$\ldots \sum_{k=-G-L_G}^{M+L_M-1} \delta_k \Delta z_{i,t-k} + \delta_{M+L_M} z_{i,t-M-L_M} + \delta_{-G-L_G-1}(-z_{i,t+G+L_G}) \ldots$$

(key part of estimating equation)

Under the linear panel model, and for general z_{it},

$$\delta_k = \begin{cases} 0 \quad \text{for} \quad k < -G \\ \sum_{m=-G}^{k} \beta_m \quad \text{for} -G \leq k \leq M \\ \sum_{m=-G}^{M} \beta_m \quad \text{for} \quad k > M. \end{cases}$$

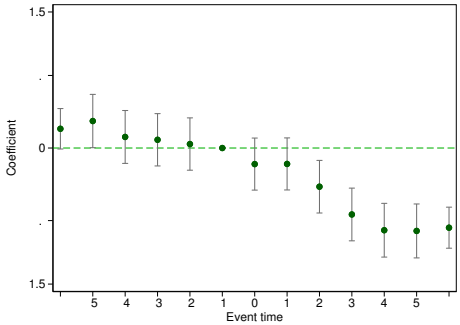
Definition of plot

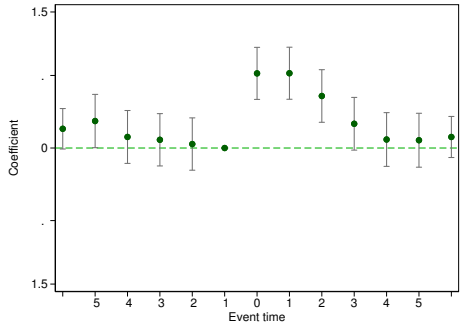


Points on plot correspond to $\{(k, \hat{\delta}_k)\}_{k=-G-L_G-1}^{k=M+L_M}$.

Suggestions

Normalization

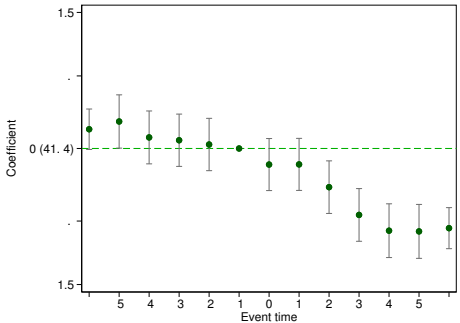


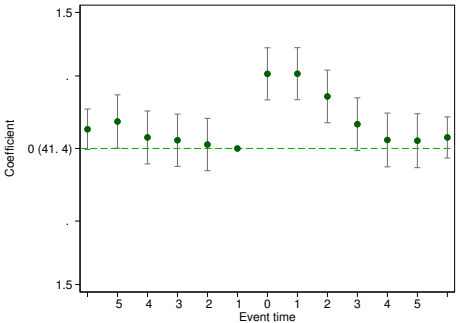


Suggestion

Normalize $\delta_{-G-1} = 0$ *in the estimating equation. (True here for* $G = 0$*.)*

Magnitude



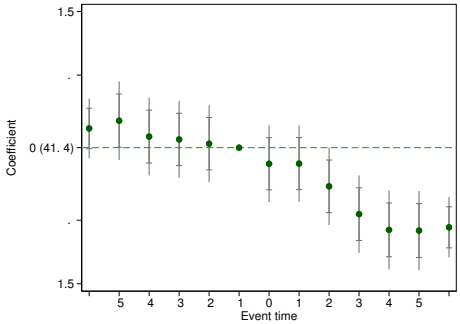


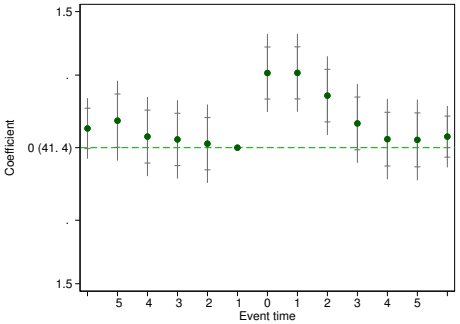
Suggestion

Include a parenthetical label showing the mean value of the dependent variable in periods corresponding to the normalized coefficient, e.g.,

$$\frac{\sum_{(i,t):\Delta z_{i,t+G+1}\neq 0} y_{it}}{|(i,t) : \Delta z_{i,t+G+1} \neq 0|}.$$

Inference

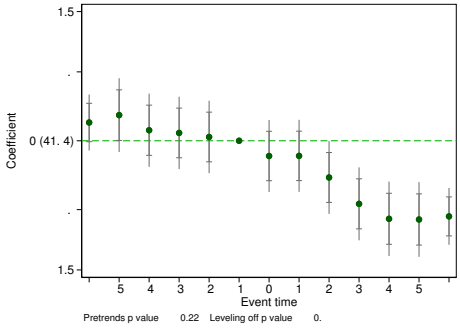


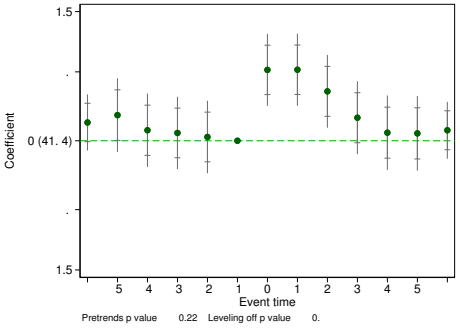


Suggestion

Add a uniform confidence band in addition to the pointwise confidence intervals.

Overidentification tests





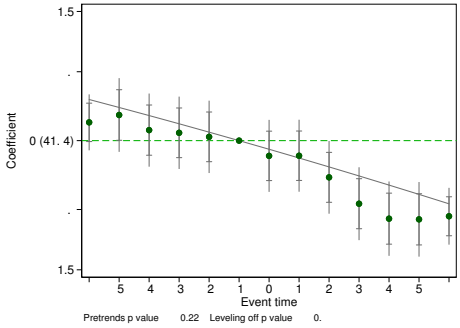
Suggestion

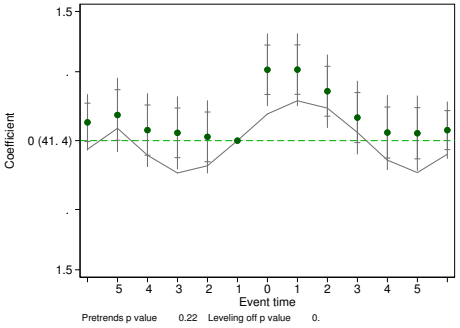
Include p-values for Wald tests of the following hypotheses:

$$H_0 : \delta_k = 0, \qquad -(G + L_G) \leq k < -G \qquad \text{(no pre-trends)}$$

$$H_0 : \delta_M = \delta_{M+k}, \qquad 0 < k \leq L_M \qquad \text{(dynamics level off)}$$

Confound paths





Suggestion

Plot the least "wiggly" confound whose event-time path is consistent with the data. Specifically, plot the polynomial with lowest-magnitude high-order coefficient among polynomials of lowest order that pass through the Wald region for δ.

Implementing suggestions with `xtevent` in Stata

- Estimation

```
xtevent y, panelvar(i) timevar(t) policyvar(z) window(5)
impute(nuchange)
```

- Event-study plot

```
xteventplot
```

- Confound dynamics

```
xteventplot, smpath(line)
```

Implementing suggestions with `eventstudyr` in R

- Estimation

```
estimates_ols <- EventStudy(
estimator = "OLS",
data = example_data,   # Use package sample data
outcomevar = "y_smooth_m",
policyvar = "z",
idvar = "id",
timevar = "t",
controls = "x_r",
pre = 0,  post = 4
)
```

- Plot

```
plt <- EventStudyPlot(estimates = estimates_ols)
plt
```

Approaches to Identification

Confound

$$y_{it} = \alpha_i + \gamma_t + q'_{it}\psi + \sum_{m=-G}^{M} \beta_m z_{i,t-m} + C_{it} + \varepsilon_{it} \qquad \text{(linear panel model)}$$

- Parameters of interest not identified unless we can say something more about the confound C_{it}
- Paper goes through a bunch of approaches; here we highlight a few

Confound is low-dimensional

Assumption 1

$$C_{it} = \lambda_i' F_t$$

with

a. $F_t = 0$ *for all* t
   - Aggregate shocks affect all units in the same way via time FE
   - Estimate with two-way fixed effects (TWFE)

b. $F_t = f(t)$ *for* $f(\cdot)$ *a known low-dimensional set of basis functions*
   - Approximating possible sources of confounding with a trend
   - Estimate with TWFE controlling for unit-specific trends

c. F_t *low-dimensional*
   - Units respond differently to common shocks
   - Estimate with interactive fixed effects, common correlated effects, or synthetic controls

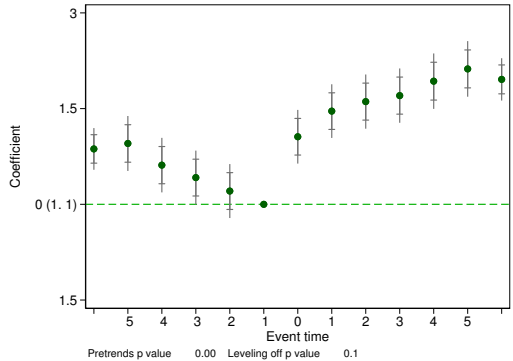
Confound can be extrapolated from pre-event period

Assumption 2

$$\mathbb{E}[C_{it}|z_i, \alpha_i, \gamma, q_i] = \tilde{\alpha}_i + \tilde{\gamma}_t + q'_{it}\tilde{\psi} + \sum_m \phi' f(m) z_{i,t-m}$$

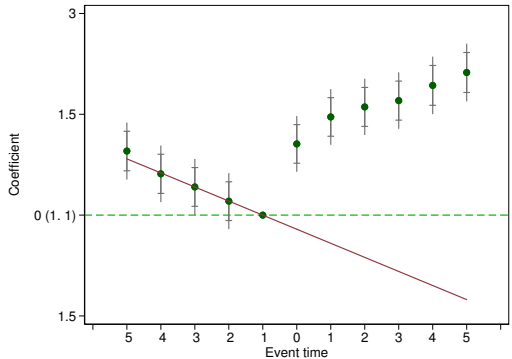
for $f(\cdot)$ a known low-dimensional set of basis functions, and $\tilde{\alpha}_i$, $\tilde{\gamma}_t$, $\tilde{\psi}$, and ϕ unknown parameters.

Basic Event-Study Plot



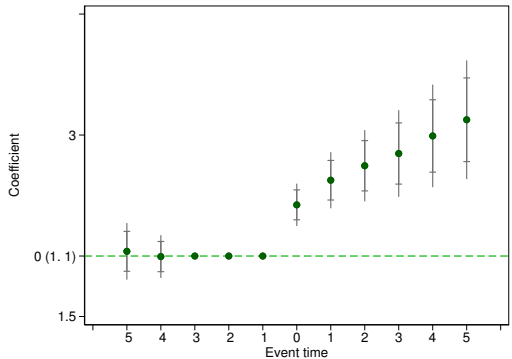

```
xtevent y, panelvar(i) timevar(t) policyvar(z) window(5)
xteventplot
```

Overlay trend




```
xtevent y, panelvar(i) timevar(t) policyvar(z) window(5)
impute(nuchange) trend(-3, saveoverlay)
xteventplot, overlay(trend)
```

Subtract extrapolated trend




```
xtevent y, panelvar(i) timevar(t) policyvar(z) window(5)
impute(nuchange) trend(-3, saveoverlay)
xteventplot, overlay(trend)
xteventplot
```

Noisy proxy with noise unrelated to policy

Assumption 3

There is an observed proxy x_{it} that obeys

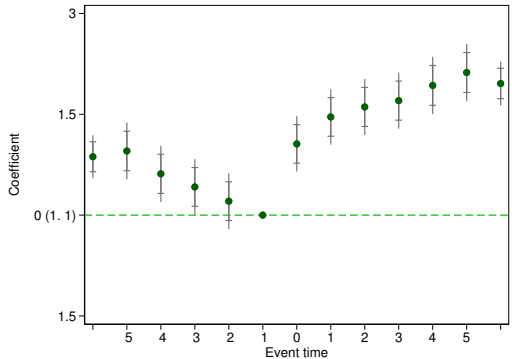
$$x_{it} = \alpha_i^x + \gamma_t^x + \psi^x q_{it} + \Xi^x C_{it} + u_{it}.$$

The unobservable u_{it} satisfies

$$\mathbb{E}[u_{it} | z_i, \alpha_i^x, \gamma_t^x, q_i] = 0,$$

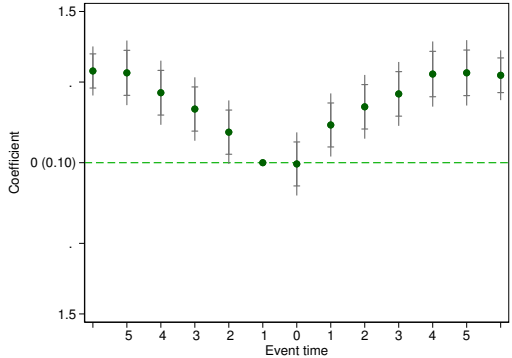
and the population projection of C_{it} on $\{z_{i,t-m}\}_{m=-G-L_G}^{M+L_M}$, q_{it}, and unit and time indicators, has at least one nonzero coefficient on $z_{i,t+m}$ for some $m > G$.

Event-study Plot for outcome



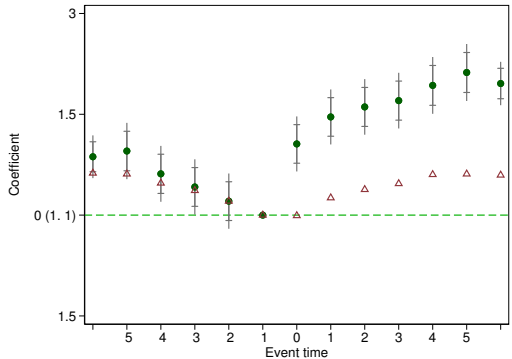

```
xtevent y, panelvar(i) timevar(t) policyvar(z) window(5)
impute(nuchange) proxy(x)
xteventplot, y
```

Event-study Plot for proxy



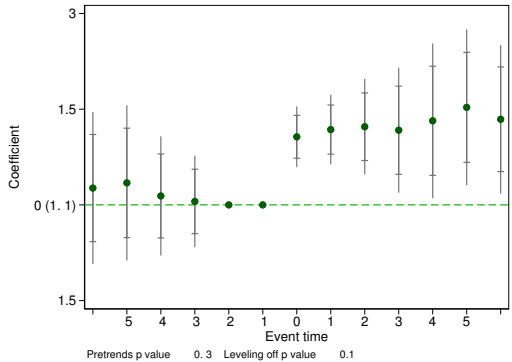

```
xtevent y, panelvar(i) timevar(t) policyvar(z) window(5)
impute(nuchange) proxy(x)
xteventplot, proxy
```

Align proxy to outcome




```
xtevent y, panelvar(i) timevar(t) policyvar(z) window(5)
impute(nuchange) proxy(x)
xteventplot, overlay(iv)
```

Subtract rescaled confound from outcome




```
xtevent y, panelvar(i) timevar(t) policyvar(z) window(5)
impute(nuchange) proxy(x)
xteventplot
```

Heterogeneous effects of the policy

- Recent literature allows the effects of the policy to differ across units
- Under staggered adoption, can allow cohort-specific policy effects with

$$y_{it} = \alpha_i + \gamma_t + q'_{it}\psi + \sum_{m=-G}^{M} \beta_{m,t^*(i)} z_{i,t-m} + C_{it} + \varepsilon_{it}$$

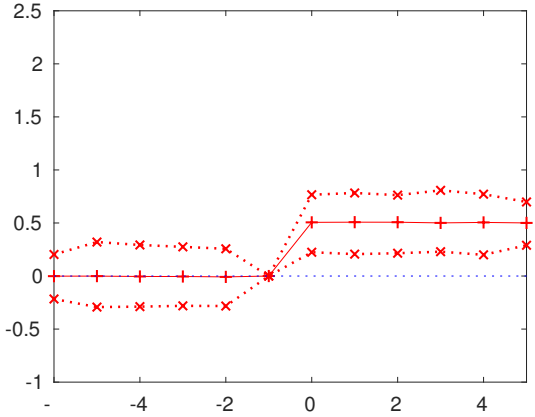
- Can then proceed with analogous restrictions on C_{it}
- Sun and Abraham: Compare each cohort to never treated units and average
- SA estimator implemented in `xtevent`

Simulations

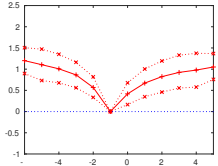
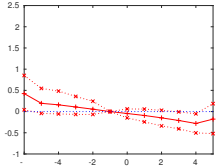
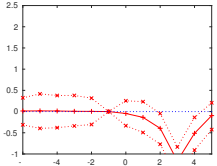
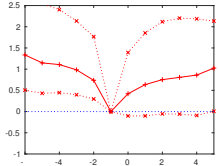
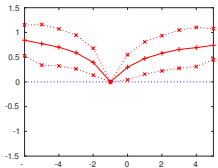
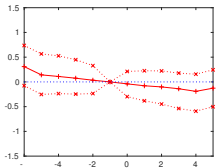
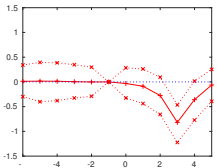
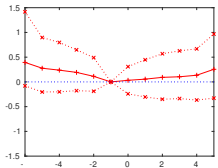
Simulation designs

- $N = 50$, $T = 40$
- Policy adopted when ($C_{i,t+P}$ + noise) crosses a threshold
- Vary P and structure of C_{it}

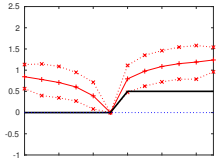
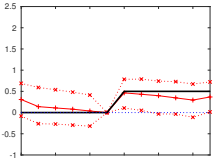
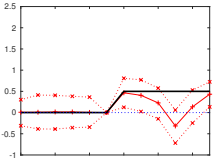
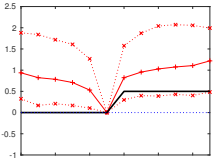
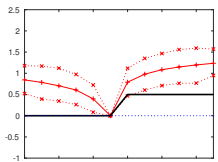
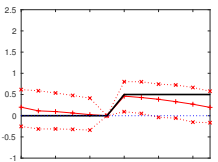
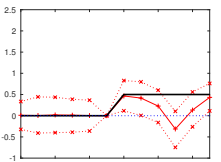
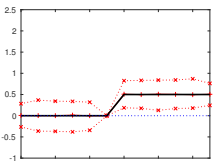
Event-study path of unconfounded outcome $y_{it} - C_{it}$



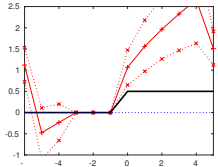
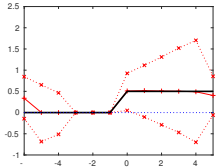
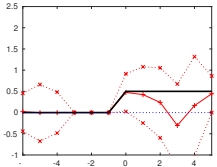
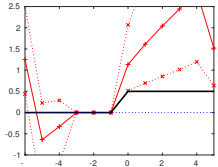
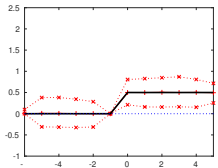
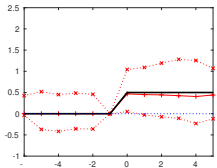
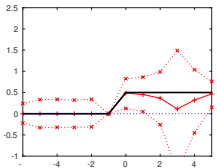
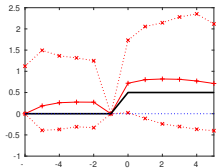
Summary of data-generating processes

Mean-rev. trend	Monotone trend	No pre-trend	Multidimensional
Confound C_{it}			
			
Proxy x_{it} if available			
			

Performance of different estimators

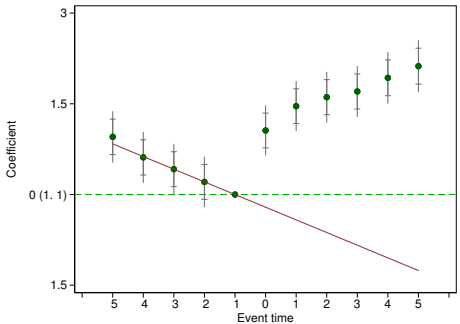
Mean-rev. trend	Monotone trend	No pre-trend	Multidimensional
Two-way fixed effects			
			
Interactive fixed-effects			
			

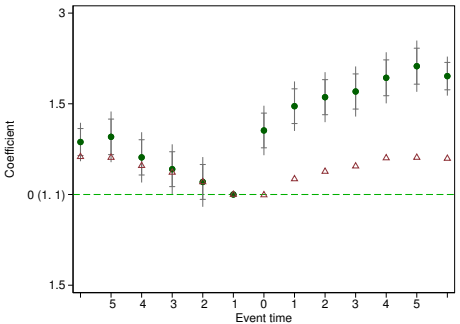
Performance of different estimators

Mean-rev. trend	Monotone trend	No pre-trend	Multidimensional
Event-time extrapolation			
			
Instrumenting for x_{it} with leads of z_{it}			
			

Takeaways

- No estimator performs well uniformly under all reasonable DGPs
- Performance of estimator cannot typically be gauged from the data at hand
- Importance of motivating modeling assumptions on economic grounds





Thank you!

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versions of paper and package